

# Anisotropic flow in Pb+Pb collisions at LHC from the quark gluon string model with parton rearrangement

J. Bleibel<sup>a</sup>, G. Burau<sup>b</sup>, C. Fuchs<sup>a</sup>

<sup>a</sup>*Institut für Theoretische Physik, Eberhard Karls Universität, Auf der Morgenstelle 14, D-72076 Tübingen, Germany*

<sup>b</sup>*Institut für Theoretische Physik, J. W. Goethe-Universität, Max-von-Laue-Straße 1, D-60438 Frankfurt am Main, Germany*

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## Abstract

We present predictions for the pseudorapidity dependence of the azimuthal anisotropy parameters  $v_1$  and  $v_2$  of baryons and inclusive charged hadrons in Pb+Pb collisions at a LHC energy of  $\sqrt{s_{NN}} = 5.5$  TeV applying a microscopic transport model, namely the quark gluon string model (QGSM) which has been recently extended for parton rearrangement and fusion processes. Pb+Pb collisions with impact parameters  $b = 2.3$  fm and  $b = 8$  fm have been simulated in order to investigate additionally the difference between central and semiperipheral configurations. In contrast to  $v_1^{\text{ch}}(\eta)$  at RHIC, the directed flow of charged hadrons shows a small normal flow alignment. The elliptic flow  $v_2^{\text{ch}}(\eta)$  turns out to be rather similar in shape for RHIC and LHC conditions, the magnitude however increases about 10 – 20% at the LHC, leading to the conclusion that the hydrodynamical limit will be reached.

*Key words:* ultra-relativistic heavy ion collisions, microscopic quark gluon string transport model, directed and elliptic flow, pseudorapidity and centrality dependence, recombination of partons

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## 1. Introduction

Ultra-relativistic heavy ion collisions have been performed within various experiments at the Relativistic Heavy Ion Collider (RHIC) in Brookhaven. Since 2000 gold on gold collisions at center of mass energies up to  $\sqrt{s_{NN}} = 200$  GeV have been investigated. After many years of operation strong experimental evidence has been accumulated, that at these energies indeed a new state of matter is created, which is qualitatively different from a hadron gas (see [1] and references therein). This new state is believed to consist of deconfined partons, as predicted by calculations within Quantum Chromodynamics (QCD) on the lattice [2,3]. It does not behave like a weakly interacting gas of partons and rather exhibits features of a strongly coupled system, a strongly coupled Quark Gluon Plasma (sQGP). The strong el-

liptic flow signal measured at RHIC [4,5,6,7,8] is one of the key observables justifying such a scenario. A strongly interacting system would imply large pressure gradients and short equilibration times [9,10], both being necessary conditions for the dynamics leading to the development of strong elliptic flow. The scaling behavior of the elliptic flow of the final hadrons with the number of constituents (see e.g. Refs. [11,12]) is a second hint towards the partonic nature of the created medium. Assuming, that the elliptic flow is to the most extent already created in the partonic phase of the collision, the observed scaling can be naturally explained as the result flow being transferred from the constituent partons to the final hadrons via parton recombination or coalescence mechanisms [13,14,15,16,17].

The newly built Large Hadron Collider (LHC) at CERN in Geneva is intended to start operation in 2008. Among the various experiments, the dedicated

Heavy Ion Program of the ALICE Collaboration [18] intends to investigate Pb+Pb Collisions at center of mass energies up to  $\sqrt{s_{NN}} = 5500$  GeV. This increase in energy of more than an order of magnitude as compared to RHIC offers the opportunity to study the properties of the strongly coupled quark gluon plasma more closely, since the energy density and lifetime of the partonic system will increase [19]. For the anisotropic flow, especially the elliptic flow, the question whether the hydrodynamical limit will be finally reached or not will be of particular interest. At RHIC energies, it has recently been concluded that this limit is reached only to a level of  $\approx 70 - 80\%$  [20,21]. This lack of perfection of the “perfect liquid”, especially at higher rapidities can be seen for example in the pseudorapidity dependence of elliptic flow [22], which cannot be described in terms of ideal hydrodynamic. So far the description of this data at RHIC has been achieved only after the inclusion of a dissipative hadronic cascade in a Hydro-Cascade hybrid model [23] and with a partonic rearrangement ansatz within the microscopic quark gluon string model [24].

LHC predictions for elliptic flow from hydrodynamical calculations as in Refs. [20,25] as well as from scaling arguments [26] show a further increase of elliptic flow, which is in line with further approach to the hydrodynamical limit, whereas a parton transport approach [27] predicts a significant smaller anisotropy parameter  $v_2$ . In the present work we apply the aforementioned quark gluon string model with parton rearrangement for lead on lead collisions at top LHC energy and two different impact parameters, namely  $b = 2.3$  fm corresponding to the mean impact parameter of the 5% most central collisions and  $b = 8$  fm as a representative impact parameter for semiperipheral collisions. Thus, we are able to present predictions for the pseudorapidity dependence of directed and elliptic flow for inclusive charged hadrons and inclusive baryons for both centralities.

## 2. QGSM with parton rearrangement

As basis for the present study serves the Monte-Carlo version of the quark gluon string model (QGSM) [28,29] which has been recently extended in order to allow for parton exchange (rearrangement) and fusion processes [24]. The standard version of the QGSM, i.e. the model without partonic rearrangements, incorporates already par-

tonic and hadronic degrees of freedom and is based on Gribov-Regge theory accomplished by a string phenomenology of particle production in inelastic hadron-hadron collisions. Thus, strings in the QGSM can be produced as a result of color exchange (Pomeron exchange) and, like in diffractive scattering, due to momentum transfer. Hard gluon-gluon scattering and semi-hard processes with quark and gluon interactions have been also incorporated in the model [30]. The cascade procedure of multiple secondary interactions of produced hadrons was implemented in order to describe hadron-nucleus and nucleus-nucleus collisions. QGSM and other string-cascade models have been successfully applied to describe directed and elliptic flow at SPS energies [31,32,33,34]. Also at RHIC, the bulk properties of elliptic flow have been fairly well reproduced within the standard version of the QGSM [35,36]. In addition it has been shown that energy densities well above critical values predicted by lattice QCD are achieved with the QGSM, and corresponding energy density profiles at proper time  $\tau = 1$  fm/c compare well with hydrodynamical assumptions for initial distributions [37].

As mentioned above, the QGSM describes particle production by the excitation and decay of open strings with different partons, namely (anti)quarks or (anti)diquarks, on their ends. Therefore it has provided a framework for the inclusion of partonic rearrangement processes which can occur in the very dense stages of a heavy ion reaction where the “hadrons” overlap and consequently are not really bound states anymore, but rather strongly correlated quark-antiquark or (anti)quark-(anti)diquark states. Please note, that in contrast to Ref. [24], we call the extension of our model *parton rearrangement* here, in order to clearly distinguish this ansatz from the well established “parton recombination” and “parton coalescence” models [14,15,16,17]. So, the idea is basically the following: Above a critical local (energy/particle) density, “hadrons” satisfying corresponding constraints are decomposed into their constituent partons which then are allowed to rearrange themselves into new “hadronic correlations”. Additionally, a quark-antiquark pair of the same flavor may annihilate during the rearrangement process with a given probability, implementing effectively a  $3 \rightarrow 2$  reaction. The probability for these annihilation processes was fixed in Ref. [24] for RHIC energies and has now been extrapolated to LHC, assuming a weak dependence on the center of mass energy of the collision:

$$P_a(\sqrt{s_{NN}}) = 0.04 \sqrt{\frac{s_{NN}}{s_{NN}^{RHIC}}}^\lambda, \quad \lambda = 0.288 \quad (1)$$

By means of that, this ansatz takes the increased likelihood for  $3 \rightarrow 2$  reactions due to the increased particle density into account. It was motivated by the so called pocket formula [38] derived within a saturation model. However, any centrality dependence of the annihilation probability has been neglected.

Partonic rearrangement or annihilation processes might very frequently happen as long as the local density of the medium is high enough. Accordingly, these rearrangement processes become more and more unimportant when the system increasingly thins out. In this spirit the model effectively emulates a medium of very strongly coupled partons, i.e. quark-antiquark and (anti)quark-(anti)diquark states, during the early times of an ultra-relativistic heavy ion collision. We want to note that this model does not create a system of “free partons”. Insofar the QGSM upgraded by the locally density dependent parton rearrangement mechanism quasi models - within its limitations - the possible dynamics of a sQGP from a microscopical point of view [24].

### 3. Anisotropic flow at LHC

Collective flow phenomena are among the main signals, which can help to reveal the formation of the sQGP in the experiment. Flow is directly linked to the equation of state of the excited matter produced in ultra-relativistic heavy ion collisions. One can subdivide the transverse collective flow into isotropic and anisotropic flow. The two most important types of anisotropic flow are characterized by the first and the second harmonic coefficients of the Fourier decomposition of the invariant azimuthal particle distribution in momentum space [39,40]:

$$E \frac{d^3N}{d^3p} = \frac{1}{\pi} \frac{d^2N}{dp_T^2 dy} \left[ 1 + 2 \sum_{n=1}^{\infty} v_n(p_T, y) \cos(n\phi) \right] \quad (2)$$

Here,  $p_T = (p_x^2 + p_y^2)^{1/2}$  is the transverse momentum,  $y$  the rapidity and  $\phi$  the azimuthal angle of a particle between its momentum and the reaction plane.

For the following study of anisotropic flow, we simulated  $Pb + Pb$  collisions at top LHC energy. For the central collisions, the number of charged hadrons at midrapidity reaches up to  $dN_{ch}/d\eta|_{\eta=0} = 3813$ , whereas in the semiperipheral case the simulation only yields  $dN_{ch}/d\eta|_{\eta=0} = 953$ .

The first harmonic coefficient  $v_1$  in Eq. (2) is called directed flow given by  $v_1 = \langle \cos(\phi) \rangle = \langle p_x/p_T \rangle$ . Its pseudorapidity dependence  $v_1(\eta)$  for inclusive charged hadrons extracted from the QGSM simulations is shown in Fig. 1. This first anisotropic compo-

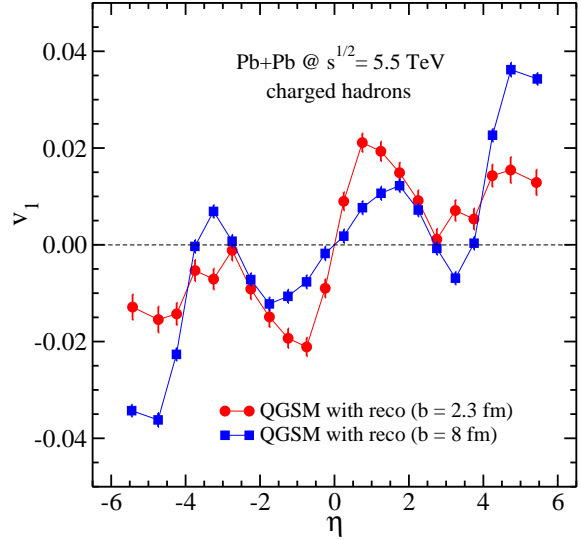


Fig. 1. Directed flow  $v_1$  of charged hadrons as a function of pseudorapidity  $\eta$  for Pb+Pb collisions at  $\sqrt{s_{NN}} = 5.5$  TeV with impact parameters  $b = 2.3$  fm and  $b = 8$  fm, respectively. Statistical errors are indicated by bars.

nent seems to show a small normal flow alignment, i.e. a positive slope  $dv_1/d\eta$ , at mid-pseudorapidity in contrast to the findings at the highest RHIC energy of  $\sqrt{s_{NN}} = 200$  GeV [41,42] where the directed flow is essentially flat and close to zero in the pseudorapidity region  $|\eta| \leq 2$ . However, for the semiperipheral collisions also  $v_1$  at LHC seems to be very small, i.e. less than 1.5%, in a broad pseudorapidity range. The structure of the directed flow remains the same for the most central collisions, however, the flow coefficient reaches higher values at even somewhat smaller rapidities. The slope is therefore even steeper in central collisions. This rather unexpected increase of  $v_1$  compared to RHIC can be understood if one considers the different viscosities in the region with  $(|\eta| < 3)$  and at higher pseudorapidities. At midrapidity, the viscosity is very low due to the small mean free path of the particles undergoing rearrangement processes. At higher rapidities, the partonic rearrangement is suppressed, therefore the viscosity is higher and due to shadowing the antiflow components become visible. A medium with very low viscosity will rather preserve its primordial flow – which is normal flow i.e.  $dv_1/d\eta > 0$ , since the in-

teractions which cause the rise of the antiflow component (shadowing) are strongly suppressed. This also explains the larger direct flow for central collisions.

Since the spectrum of final hadrons is dominated by pions, we separately show only the directed flow of all baryons in Fig. 2. Interestingly, the baryonic  $v_1$  shows a negative slope  $dv_1/d\eta$ , conventionally called antiflow, for at least the semiperipheral collisions. At higher values of  $|\eta|$ , the directed flow of all baryons is rather large. The situation is not so clear in more central reactions. Here, the magnitude of the directed flow is compatible with zero at least in the pseudorapidity region  $|\eta| < 1.5$ , but then peaking at around  $|\eta| \approx 2 - 3$ . However, the statistical errors are still too large to draw a definite conclusion. In both cases the flow of baryons shows the same behavior as in our previous study at RHIC energies [35]. It is less affected by the rearrangement processes which play only a minor role for baryons.

Next we investigate the pseudorapidity dependence of the elliptic flow  $v_2(\eta)$ , i.e. the second harmonic coefficient of the Fourier decomposed invariant azimuthal distribution of produced particles given by Eq. (2). This anisotropic flow component is determined by  $v_2 = \langle \cos(2\phi) \rangle = \langle (p_x/p_T)^2 - (p_y/p_T)^2 \rangle$ . The QGSM simulation results for Pb+Pb collisions at  $\sqrt{s_{NN}} = 5.5$  TeV with impact parameters  $b = 2.3$  fm and  $b = 8$  fm are depicted in Fig. 3. The crucial result of an analogous study for Au+Au collisions at RHIC with the

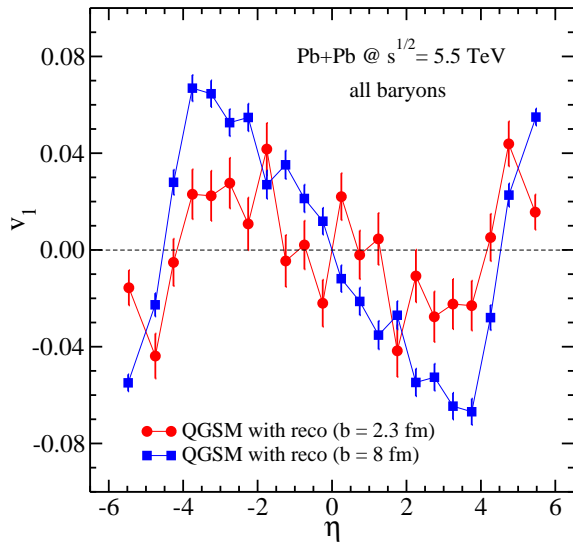


Fig. 2. Pseudorapidity distribution of  $v_1$  for baryons only. The simulated collisions are the same as in Fig. 1.

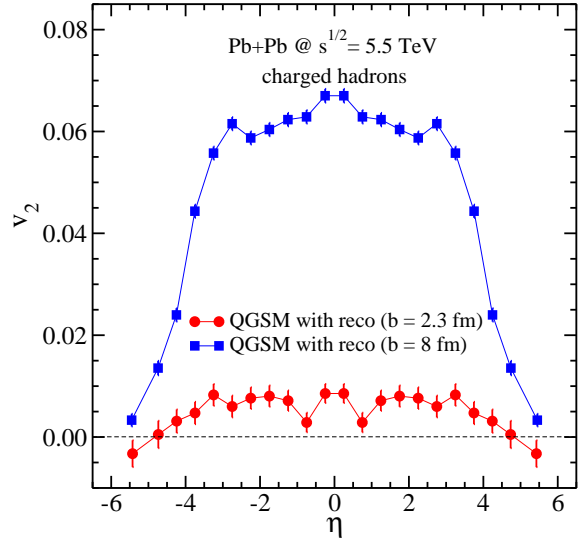


Fig. 3. Elliptic flow  $v_2$  vs. pseudorapidity  $\eta$  of inclusive charged hadrons from Pb+Pb simulations with the QGSM extended by parton rearrangement processes. The error bars denote statistical uncertainties.

QGSM [24], which has been extended by a locally density dependent partonic rearrangement mechanism in order to model effectively the dynamics of a very strongly coupled quark plasma at high particle densities, was that the shape of the anisotropy parameter  $v_2(\eta)$  of final charged hadrons is intimately related to the collision dynamics. It turned out within this microscopic investigation that fast equilibration due to parton rearrangement and fusion processes, which occur in the very dense medium created in ultra-relativistic heavy ion reactions during the early stages, are necessary in order to obtain  $v_2(\eta)$  profiles which are peaked at midrapidity as seen in the RHIC data. Hence it might be no surprise to find a very similar qualitative behavior for LHC conditions with a much higher collision energy.

Here, the extended QGSM predicts for semiperipheral collisions also a strong in-plane alignment of  $v_2$  with a peak at  $|\eta| \approx 0$  and a steady decrease for larger values of  $|\eta|$ , but the total distribution is of course broader compared to  $v_2(\eta)$  at RHIC. Furthermore, the maximum value of the elliptic flow around midrapidity of  $v_2(\eta = 0) = 6.7\%$  is significantly higher than at the highest RHIC energy: The impact parameter  $b = 8$  fm corresponds to a centrality of the collision of  $\approx 25\%$ , or a mean number of participants of  $N_{part} = 180$ . Such a maximum value of ( $p_t$ -integrated)  $v_2$  is not observed at RHIC even in more semiperipheral collisions, i.e.  $25 - 50\%$ ,  $N_{part} \approx 111$  [7]. A simulation with the extended QGSM at max-

imum RHIC energy, analogous to Ref. [24] but with the same number of participants as above, yielded a value of  $v_2(\eta = 0) = 5.9 \pm 0.2\%$ . The predicted maximum value at LHC is therefore about 10 – 20% greater than at RHIC. This result is in line with the assumption, that the hydrodynamical limit was not reached at RHIC. In the hydrodynamical regime the elliptic flow would scale with the eccentricity  $\epsilon$  of the overlap of the colliding nuclei, or  $v_2/\epsilon = \text{const.}$ . However, such a scaling cannot be found (see e.g. Refs. [20,21]), the hydrodynamical limit was therefore reached only up to 70 – 80%. The predicted increase of elliptic flow at LHC can in this context be interpreted as a much closer approach to the hydro-limit. It has been shown that microscopic transport calculations can indeed, at least in 2D, approach this limit [43]. In our model, the convergence towards the hydrodynamical limit can be nicely explained by the effect of the viscosity of the medium on  $v_2$ : due to the many partonic rearrangements the mean free path in the medium of produced particles is reduced. This also lowers the viscosity, and by having more and more rearrangement processes due to the increased energy- and particle density at LHC, the mean free path and thus the viscosity may actually be minimized. This would lead to a maximum value for the elliptic flow [44].

A second argument for approaching the hydrodynamical limit can be drawn from the ratio  $v_4/v_2^2$ . It has been argued in Refs. [44,45] that this ratio probes the degree of equilibration of the produced matter, leading to  $v_4/v_2^2 \rightarrow 1/2$  in the hydrodynamical limit. Preliminary results for this observable, averaged for  $0.15 < p_t < 2.0$  GeV and  $|\eta| < 4$ , yield for charged hadrons a value of  $v_4/v_2^2 \approx 0.76 \pm 0.24$ . Despite the large errors, the decrease from the measured  $v_4/v_2^2 = 1.2$  at RHIC [42] and  $v_4/v_2^2 \approx 1.29 \pm 0.27$  as result of a similar analysis at top RHIC energy with the same number of participants and based on the data of Ref. [24], supports a further approach of the hydrodynamical limit and is in line with transport calculations by Ko et al. [46].

The centrality dependence of  $v_2(\eta)$  is qualitatively similar for RHIC and LHC conditions: The elliptic flow is large for peripheral reactions and rather small for central ones. The result for the central collisions yields a rather flat distribution of  $v_2$  over a broad centrality range with values around  $v_2 \approx 1\%$ . Whether the distribution is peaked or not cannot be decided due to the limited statistics.

For the sake of completeness, Fig. 4 shows the pseudorapidity distributions of  $v_2$  for all baryons

only, which have been extracted from the aforementioned Pb+Pb collisions at  $\sqrt{s_{NN}} = 5.5$  TeV with an impact parameter of  $b = 8$  fm. The distribution for

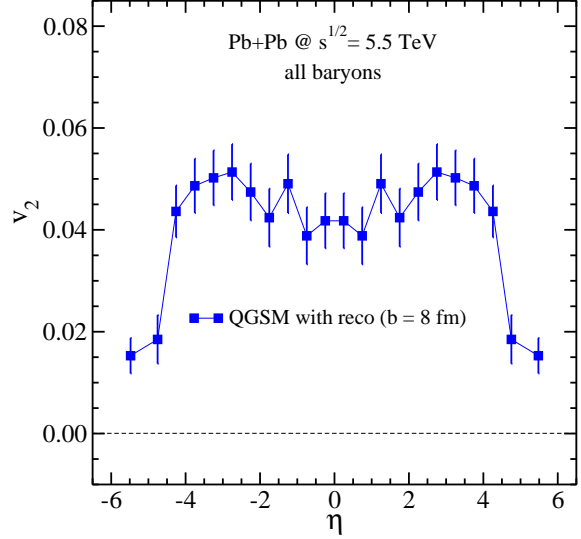


Fig. 4. The same like in Fig. 3, but for inclusive charged nucleons only.

all baryons shows a different structure. In contrast to the single peaked pseudorapidity distribution of elliptic flow of charged hadrons here two peaks at pseudorapidities  $|\eta| \approx 3$  are predicted. The values of  $v_2$  at higher rapidities are quite similar to the result with all charged hadrons. At midrapidity, the flow is significantly smaller.

#### 4. Summary and conclusions

For this survey, we have analyzed simulated Pb+Pb collisions at a center of mass energy of  $\sqrt{s_{NN}} = 5.5$  TeV with impact parameters  $b = 2.3$  fm and  $b = 8$  fm applying a microscopic string-cascade transport model, namely the quark gluon string model (QGSM), which has been recently extended for locally density dependent parton rearrangement and fusion processes in order to emulate effectively a medium of very strongly correlated partons, i.e. quark-antiquark and (anti)quark-(anti)diquark states, and its dynamics. Predictions for the pseudorapidity dependence of the azimuthal anisotropy parameters  $v_1(\eta)$  and  $v_2(\eta)$  of nucleons and inclusive charged hadrons for central and semiperipheral collision configurations have been presented. The directed flow of charged hadrons shows a small normal flow alignment at midrapidity for both the central and semiperipheral reactions

in contrast to the findings at RHIC, but it is rather small at all, i.e. less than 2%. The elliptic flow of charged final hadrons turns out to be large for semiperipheral Pb+Pb collisions with a maximum value of about 6.7% and small for the central ones. The second anisotropy parameter seems to be rather similar in shape for RHIC and LHC conditions, the maximum value at midrapidity however has been predicted to increase by about 10 – 20%. Therefore it has been concluded that the hydrodynamical limit is quasi reached.

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